Lesson 17 Part 1: Introduction
Solve Problems with Inequalities

CCLS
7.EE.B.3
7.EE.B.4b

You've learned how to solve two-step equations. Take a look at this problem.

Mr. Thomas brings $100 to a fundraiser. He wants to leave the event with at least $50 in his pocket. Guests at the fundraiser buy raffle tickets for several different prizes. Each raffle ticket costs $2.50. How many raffle tickets can Mr. Thomas buy and still leave with at least $50 in his pocket?

Explore It

Use math you already know to solve the problem.

- How much money does Mr. Thomas have at the start of the fundraiser? _____
- Let t = the number of tickets bought. Write an expression to show how much it costs to buy t tickets. _____
- Use the expression above to write a different expression that shows how much money Mr. Thomas would have left after buying t tickets. _____
- Suppose Mr. Thomas buys 25 tickets. How much money would he have left? Is this at least $50? Show your work.

  _____
- Suppose Mr. Thomas buys 20 tickets. How much money would he have left? Is this at least $50?

  _____
- What is the greatest number of tickets Mr. Thomas can buy and still have at least $50 left? Explain. _____

  _____
- Could Mr. Thomas buy fewer than 20 tickets? Explain.

  _____

  _____
- Fill in the blank. Mr. Thomas could buy any number of tickets that is _____ or fewer.

Find Out More

You can solve the problem on the previous page by writing and solving an inequality.

(starting amount) − (ticket price) • (number of tickets) is at least (amount left)

$100 \quad - \quad 2.5 \quad \cdot \quad t \quad \geq \quad 50$

Solving a two-step inequality is similar to solving a two-step equation. But, when you multiply or divide each side of the inequality by a negative number, you reverse the inequality symbol.

$$100 - 2.5t \geq 50$$

$$100 - 100 - 2.5t \geq 50 - 100$$

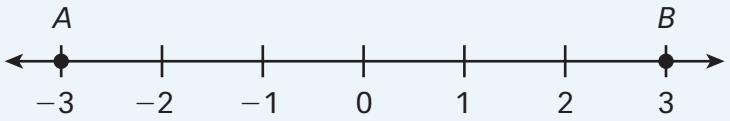
$$-2.5t \geq -50$$

$$\frac{-2.5t}{-2.5} \leq \frac{-50}{-2.5} \quad \text{(Reverse the symbol.)}$$

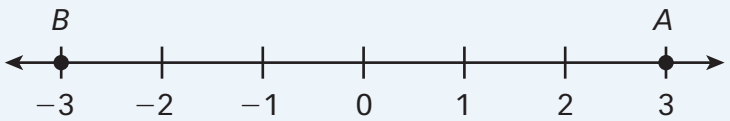
$$t \leq 20$$

He can buy 20 or fewer tickets.

Let's use integer inequalities to examine why the symbol is reversed.
A is -3 and B is 3, so, $A < B$. On the number line, A is to the left of 0 and B is to the right.



Now divide both A and B by -1 and compare the values. A becomes 3 and B becomes -3.
Now $A > B$ and A is to the right of 0 and B is to the left.



After dividing by a negative number, the quotient is on the opposite side of 0, which means the symbol is reversed.

Reflect

1 What is the effect of multiplying both sides of an inequality by a negative number? Explain, and give an example.

Read the problem below. Then explore different ways to solve a two-step inequality.

Chang has at most \$60 to spend on socks and sneakers. He finds a pair of sneakers that he likes for \$36. If socks are \$3 per pair, how many pairs of socks could Chang buy?

Model It

You can write and solve an inequality to understand the problem.

The price of the sneakers and socks combined must be \$60 or less.

price of sneakers $+$ 3 • p pairs socks must be $<$ or $=$ \$ Chang has

$36 + 3p \leq 60$

$36 - 36 + 3p \leq 60 - 36$

$3p \leq 24$

$\frac{3p}{3} \leq \frac{24}{3}$

$p \leq 8$

Model It

You can graph the solution set on a number line.

The inequality $p \leq 8$ means all values less than or equal to 8. However, in this situation, only whole numbers make sense. You cannot buy a fraction of a pair of socks, or a negative number of pairs of socks.

Connect It

Now you will look at the solution and graph to analyze and interpret the inequality.

2 What does the inequality statement in Model It mean?

3 List the steps for solving this two-step inequality, using the terms constant and coefficient.

4 Explain why the numbers between each whole number are not a part of the solution set.

5 According to the graph, what is the complete solution set for the inequality?

Why are −1 and −2 not in the solution set? _____

6 How do you graph the solution set of an inequality when that set includes only whole numbers? _____

Try It

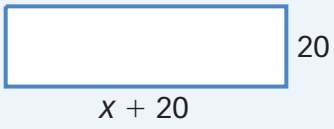
Use what you just learned about inequalities to solve these problems.

7 Students in the garden club are planting a spring flower garden in the town square. They have already spent $80 of the $200 budget. Write and solve an inequality to show how many $30 packs of bulbs they can buy. _____

8 Draw a number line and graph the solution set.

Read the problem below. Then explore different ways to solve a multi-step inequality.

Mrs. Sanchez is building a laundry room in the basement of the apartment building she owns. Given the layout of the basement, she wants the width of the room to be 20 feet and the length to be longer than the width. If she wants the area of the room to be more than 500 square feet, what could be the length? Look at the diagram Mrs. Sanchez drew.



Model It

You can write and solve an inequality to find *x*.

The product of the width and length must be greater than 500 square feet.

width	•	length	more than	area
↓		↓	↓	↓
20	•	$(20 + x)$	$>$	500

$$20(20 + x) > 500$$

$$400 + 20x > 500$$

$$400 - 400 + 20x > 500 - 400$$

$$20x > 100$$

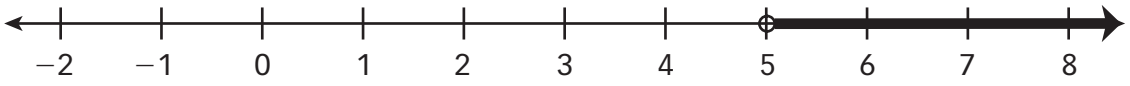
$$\frac{20x}{20} > \frac{100}{20}$$

$$x > 5$$

Model It

You can graph the solution set for *x* on a number line.

In this problem situation, the numbers in the solution can be whole numbers, decimals, or fractions. You can measure to a fraction of a foot or inch and that measurement could be used to solve the problem. So, the graph shows a solid line.



Connect It

Now you will look at the solution and graph to analyze and interpret the inequality.

9 Describe in words what the inequality statement in the first Model It means. _____

10 What do you do with any value for x in the solution set to find the length? Explain.

11 Look at the graph of the solution set. Why is there an open circle on the 5? Why is there a solid line instead of just circles on the whole numbers?

12 The numbers on the number line end at 8. The arrow shows that numbers beyond 8 are also in the solution set. Given the problem situation, do you think the solution set can extend forever, or will there be a limit? Explain. _____

13 In words, describe what the solution set is and what it means in context of the problem.

Try It

Use what you just learned about inequalities to solve this problem. Show your work on a separate sheet of paper.

14 Solve $8(12 - \frac{1}{4}x) \geq 82$ and graph the solution set on a number line.

Study the model below. Then solve problems 15–17.

Student Model

At the beginning of baseball season, Coach Thorne takes inventory of the team equipment to see what he needs. He counts 24 baseballs, but he needs to start off the season with at least 100 balls. The balls that he uses are sold in packages of 4. How many packages could the coach buy?

Look at how you could show your work by solving an inequality.

$$24 + 4x \geq 100$$
$$24 - 24 + 4x \geq 100 - 24$$
$$4x \geq 76$$
$$\frac{4x}{4} \geq \frac{76}{4}$$
$$x \geq 19$$

Solution: **Coach Thorne could buy 19 or more packages.**

The inequality shows that the 24 balls the coach has plus x packages of 4 balls each must be greater than or equal to 100 balls.

Pair/Share

How would the inequality change if the problem said "more than 100 balls?"

15 Market and More is having a cereal sale. Every box of cereal is $0.60 off the regular price. Jane has $10 and she wants to buy 4 boxes of the same cereal. She uses the inequality below to determine the regular price of cereal that she can afford. Solve the inequality and explain what the solution means.

$4(r - 0.6) \leq 10$, where r = regular price

Show your work.

Four times the sale price of the box has to be ten dollars or less.

Pair/Share

Try some different prices in the solution set and discuss the results.

Solution: _____

16 Solve the inequality and graph the solution set on a number line.

$200 - 4.5x \leq 20$

Show your work.

Solution: _____

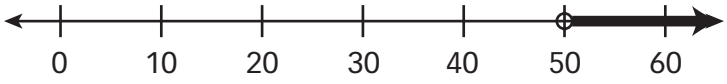
What happens when you multiply or divide both sides of an inequality by a negative number?

Pair/Share

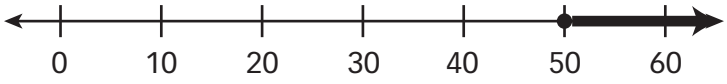
Talk about situations that this inequality might represent.

17 Greg calculated that he had to drive at least 50 miles per hour on the highway to get to his destination in the time that he has. Which number line shows the solution set to this inequality?

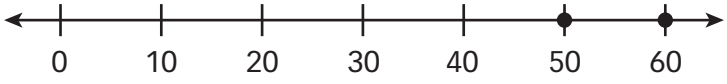
A



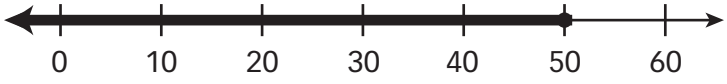
B



C



D



Jess chose **C** as the correct answer. How did she get that answer?

_____

_____

_____

_____

Think about what "at least" means in the context of the problem.

Pair/Share

Discuss what the graph in **C** means.

Solve the problems.

1 You need to reverse the symbol to solve which inequality?

A $4(5 - y) \geq 80$

B $2(y - 3) \geq 8$

C $3y - 4 < 11$

D $-\frac{3}{4} + 6y > \frac{1}{4}$

2 The number line shows the solution set to which inequality?

A $12 + 4x > 22$

B $3x + 2 \geq 17$

C $12 + 4x \geq 22$

D $4x - 12 \leq 22$

3 Sally wants to spend no more than $16 on school supplies. The table shows how much each item costs at the school store. No tax is charged.

Item	Price
Loose leaf	$2.55/package
Pen	$1.22/pen
Binder	$3.99/binder
Eraser	$0.67/eraser

Which combination of items can Sally buy? Select all that apply.

A 6 loose leaf and 6 pens

B 8 pens and 8 erasers

C 3 pens and 4 binders

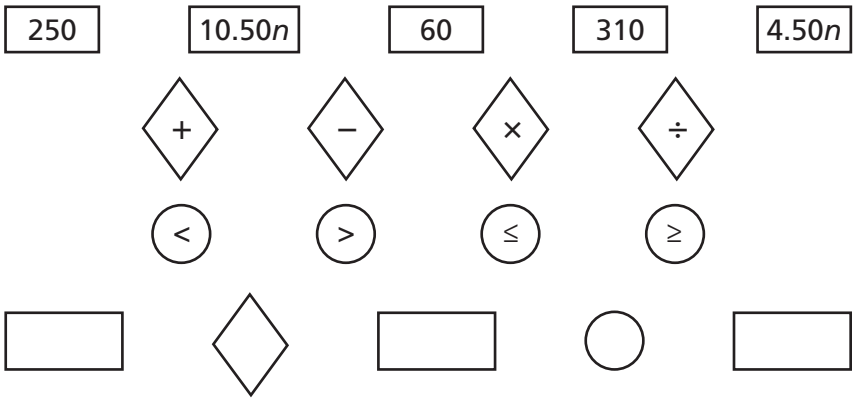
D 4 loose leaf and 8 erasers

E 2 loose leaf and 3 binders

F 3 binders and 6 erasers

4 A salesperson is paid \$60 per week plus \$4.50 per sale. This week, the salesperson wants to earn at least \$250. How many sales, n, must the salesperson make in order to meet that goal?

Write in each box the appropriate given number, operation, or symbol that creates an inequality to determine the minimum number of sales, n, the salesperson must make.



5 Raj has a \$25 budget to spend on decorations for a party. He has already spent \$18.60. He now wants to get some helium balloons that cost \$0.80 each. Write and solve an inequality to show the number of balloons that Raj could buy.

Show your work.

Answer Raj can buy _____ balloons.

6 Graph the solution set for the inequality in problem 5. Describe what the solution means and how the number of balloons bought will affect the amount in the budget.

Show your work.

Answer _____

_____

_____

_____

✓ Self Check *Go back and see what you can check off on the Self Check on page 125.*