## Lesson 1 Part 1: Introduction Properties of Integer Exponents

In the past, you have written and evaluated expressions with exponents such as $5^{3}$ and $x^{2}+1$. Now, take a look at this problem.

Multiply: ( $3^{3}$ ) ( $3^{4}$ )

## Q Explore It

## Use the math you know to answer the questions.

What do the expressions $\left(3^{3}\right)$ and $\left(3^{4}\right)$ have in common?

- Write a multiplication expression without exponents that is equivalent to $3^{3}$. $\qquad$
How many factors of 3 did you write? $\qquad$
- Write a multiplication expression without exponents that is equivalent to $3^{4}$. $\qquad$
- How many factors of 3 did you write?
- Write a multiplication expression without exponents that is equivalent to $\left(3^{3}\right)\left(3^{4}\right)$.
- How many factors of 3 did you write? $\qquad$
- Write an expression with exponents to complete this equation: $\left(3^{3}\right)\left(3^{4}\right)=$ $\qquad$
- What is the relationship between the exponents of the factors and the exponent of the product in your equation?
$\qquad$
- Use words to explain how to multiply $\left(3^{3}\right)\left(3^{4}\right)$.


## Q Find Out More

You have seen one example of how to multiply powers with the same base. Here are two more:

$$
\begin{aligned}
& \left(5^{8}\right)\left(5^{5}\right)=5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5=5^{8+5}=5^{13} \\
& \left(x^{6}\right)\left(x^{2}\right)=x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x=x^{6+2}=x^{8}
\end{aligned}
$$

In general, for the product of powers with the same base, $\left(n^{a}\right)\left(n^{b}\right)=n^{a+b}$, where $n \neq 0$.
You can also use the meaning of exponents to divide powers with the same base.
Divide $\frac{4^{12}}{4^{5}}$.

$$
\frac{4^{12}}{4^{5}}=\frac{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4}{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4} \quad 4^{12} \text { is twelve } 4 \mathrm{~s} \text { multiplied together. }
$$

$$
=\frac{4}{4} \cdot \frac{4}{4} \cdot \frac{4}{4} \cdot \frac{4}{4} \cdot \frac{4}{4} \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \quad 4^{5} \text { is five } 4 \mathrm{~s} \text { multiplied together. }
$$

$$
=1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \quad \text { Any non-zero number divided by itself is } 1 .
$$

$$
=4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \quad \text { Seven } 4 s \text { multiplied together is } 4^{7}
$$

$$
=4^{7}
$$

So, $\frac{4^{12}}{4^{5}}=4^{7}$. What is the relationship between the exponents of the dividend, divisor, and quotient? The exponent of the quotient is the exponent of the dividend minus the exponent of the divisor. $12-5=7$.
In general, for the quotient of two powers with the same base, $\frac{n^{a}}{n^{b}}=n^{a-b}$, where $n \neq 0$.

## Reflect

1 Explain why $\frac{5^{10}}{5^{2}}$ equals $5^{8}$.
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## Read the problem below. Then explore how to find the product of powers with the same base and the same exponent.

Simplify: $\left(3^{2}\right)^{4}$

## Model It

## You can write it another way.

$\left(3^{2}\right)^{4}=$ means 3 squared, multiplied as a factor 4 times.
$\left(3^{2}\right)^{4}=3^{2} \cdot 3^{2} \cdot 3^{2} \cdot 3^{2}$
$\left(3^{2}\right)^{4}$ is the product of 4 powers, each with the same base (3) and the same exponent (2).

## Q Solve It

You can apply the associative property of multiplication.

$$
\begin{array}{rlrl}
\left(3^{2}\right)^{4} & =3^{2} \cdot 3^{2} \cdot 3^{2} \cdot 3^{2} & \left(3^{2}\right)^{4} \text { is the product of four } 3^{2} s \text { multiplied together. } \\
& =\left(3^{2} \cdot 3^{2}\right)\left(3^{2} \cdot 3^{2}\right) & & \text { Apply the associative property of multiplication. } \\
& =\left(3^{4}\right)\left(3^{4}\right) & & \text { This is the product of powers with the same bases. } \\
& =3^{4+4} & & \text { Add the exponents. } \\
& =3^{8} & &
\end{array}
$$

## Connect It

## Now you will explore the concept from the previous page further.

2 Simplify: $\left(3^{2}\right)^{4}=$ $\qquad$
3 Describe the relationship between the exponents of $\left(3^{2}\right)^{4}$ and the exponent of $3^{8}$.

4 Complete these examples of products of powers that have the same base and the same exponent.

$$
\begin{aligned}
& \left(5^{8}\right)^{6}=5^{8} \cdot 5^{8} \cdot 5^{8} \cdot 5^{8} \cdot 5^{8} \cdot 5^{8}=5^{8+8+8+8+8+8}=5^{8 \cdot 6}= \\
& \left(953^{7}\right)^{3}=953^{7} \cdot 953^{7} \cdot 953^{7}=953^{7+7+7}=953^{7 \cdot 3}=
\end{aligned}
$$

5 In general, for a product of powers that have the same base and the same exponent, $\left(n^{a}\right)^{b}=$ $\qquad$ where $n \neq 0$.

Now look at how to simplify a product of powers when the bases are different and the exponents are the same.

Simplify: $\left(2^{3}\right)\left(4^{3}\right)$

6 Write an expression without exponents that is equivalent to $\left(2^{3}\right)\left(4^{3}\right)$.
7 Apply the associative and commutative properties of multiplication to write your expression as the product of groups of $2 \cdot 4$. $\qquad$
8 How many groups of $2 \cdot 4$ do you multiply together to get $\left(2^{3}\right)\left(4^{3}\right)$ ? $\qquad$
9 Complete this equation: $\left(2^{3}\right)\left(4^{3}\right)=(2 \times 4)^{\square}=\square^{3}$
10 In general, for a product of powers that have different bases and the same exponent, $\left(a^{n}\right)\left(b^{n}\right)=$ $\qquad$ , where $a \neq 0$ and $b \neq 0$.

## Try It

Use what you just learned to solve these problems. Write your answers using exponents.

11 Simplify: $\left(2^{18}\right)^{8}=$ $\qquad$
12 Simplify: $\left(4^{9}\right)\left(25^{9}\right)=$ $\qquad$ or $\qquad$

## Read the problem below. Then explore simplifying expressions with exponents equal to zero.

```
Simplify: \(5^{0}\)
```


## Model It

## You can write it another way.

It doesn't make sense to ask yourself, "What is zero 5s multiplied together?" We will need to approach this problem another way.
So far, you have worked with powers where the exponents are counting numbers (1, 2, 3, . . ). The rules for working with powers are the same when the exponent is 0 .

You have seen that when you multiply powers with bases that are the same you add the exponents.

$$
\left(5^{0}\right)\left(5^{4}\right)=5^{0+4}=5^{4}
$$

## Q. Solve It

You can apply the identity property of multiplication.
You know that 1 times any expression is equivalent to that expression by the identity property of multiplication.
(1) $\left(5^{4}\right)=5^{4}$

Because (1) $\left(5^{4}\right)=5^{4}$
and $\left(5^{0}\right)\left(5^{4}\right)=5^{4}$,
$5^{0}$ must therefore be equal to 1 .

## Q Connect It

## Now you will explore the concept from the previous page further.

13 Simplify: $5^{0}=$ $\qquad$
14 Complete these examples:

$$
12^{0}=\square=1
$$

$(-7)^{0}=$ $\qquad$
15 In general, for a power where the exponent is equal to $0, n^{0}=$ $\qquad$ where $n \neq 0$.

The rules for products of powers also apply when the exponent is a negative integer.
16 Complete this equation: $\left(6^{5}\right)\left(6^{-5}\right)=6 \square=$ $\qquad$
17 You already know that a number times its reciprocal equals 1 . For example, $3 \cdot \frac{1}{3}=\frac{3}{3}=1$.
Now complete this equation: $6^{5} \cdot \frac{1}{6^{5}}=$ $\qquad$ $=$

18 Since $6^{5} \cdot 6^{-5}=$ $\qquad$ and $6^{5} \cdot \frac{1}{6^{5}}=$ $\qquad$ , then $6^{-5}=$ $\qquad$
19 Complete these examples:
$10^{-6}=$ $\qquad$
$(-34)^{-7}=$ $\qquad$
$\qquad$

$$
=\frac{1}{142^{13}}
$$

20 In general, for a power where the exponent is a negative integer, $n^{-a}=$ $\qquad$ where $n \neq 0$.

## Try It

Use what you just learned to solve these problems. Write your answers using exponents where appropriate.

21 Simplify: $455^{\circ}=$ $\qquad$
22 Simplify: $19^{-4}=$ $\qquad$

In this problem, you have to apply more than one rule of working with exponents.


## Pair/Share

If $x$ and $a$ are counting numbers, is $x^{-a}$ less than or greater than 1? Explain.

Remember the order of operations. Simplify the expression within the parentheses first.


## Pair/Share

Does $5^{9} \cdot 6^{7}=(30)^{16}$ ? Justify your answer.

## Study the student model below. Then solve problems 23-25.

Simplify: $2^{4} \cdot 2^{-7}$
Look at how you could show your work.

$$
\begin{array}{ll}
2^{4} \cdot 2^{-7} & \text { product of powers with equivalent bases } \\
=2^{4+(-7)} & \text { add exponents } \\
=2^{-3} & \text { power with a negative integer exponent } \\
=\frac{1}{2^{3}} & \text { reciprocal with positive exponent } \\
\text { Solution: } & 2^{4} \cdot 2^{-7}=\frac{1}{2^{3}}
\end{array}
$$

23 Simplify: $\left(3^{2} \cdot 4^{2}\right)^{5}$
Show your work.
$\qquad$

24 Simplify: $9^{-8} \cdot \frac{1}{9^{3}}$. Write your answer with a positive exponent.
Show your work.

Solution: $\qquad$

25 Which expression is equivalent to $\frac{45^{-3}}{45^{3}}$ ?
A $45^{-1}$
B $45^{\circ}$
C $\frac{1}{45^{6}}$
D $45^{6}$
Isaac chose A as the correct answer. How did he get that answer?
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## Solve the problems.

1 Which expression is equivalent to $\left(-4^{-5}\right)^{0}$ ?
A 1

B $\quad(-4)^{5}$
C $\frac{1}{(-4)^{5}}$
D $\quad \frac{1^{5}}{-4}$

2 Which expression is equivalent to $\frac{\left(7^{2}\right)^{5} \text { ? }}{7^{-6}}$ ?
A 7
B $\quad 7^{4}$
C $\quad 7^{13}$
D $\quad 7^{16}$

3 Which expression is equivalent to $\frac{1}{49}$ ? Select all that apply.
A $7^{-1} \times 7^{-1}$
B $\quad 7^{8} \times 7^{-6}$
C $\quad 7^{-5} \times 7^{3}$
D $\quad 7^{7} \times 7^{-9}$
E $\quad 7^{-2} \times 7^{4}$
$4 \quad$ Write $16^{8}$ as a power with a base of 4.

5 Look at the equations below. Choose True or False for each equation.
A $\quad 2^{4} \times 3^{4}=4^{6}$
 True $\square$ False

B $\quad 5^{2} \div 5^{3}=\frac{1}{5}$
 True $\square$ False

C $\left(6^{3}\right)^{4}=\left(6^{4}\right)^{3}$
 True
 False

D $\frac{3^{2}}{3^{7}}=3^{2} \times 3^{-7}$
 True

False
E $\quad \frac{8^{0}}{8^{-4}}=8^{-4}$
 True $\square$ False

F $\quad 4^{10} \div 4^{5}=4^{2}$ $\square$ True

6 Write each of these numbers as the product of a whole number and a power of 10. Then describe the relationship between place value and exponents.

$$
\begin{aligned}
& 3,000= \\
& 300= \\
& 30= \\
& 3= \\
& 0.3= \\
& 0.03= \\
& 0.003= \\
&
\end{aligned}
$$

